

Scaling of near-wall streaks in turbulent channel flow at moderate Reynolds number

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Direct numerical simulations of turbulent channel flow are used to examine how the spanwise spacing of near-wall streaks varies with friction Reynolds number in the range $Re_\tau = 180$ –1000. The streak spacing, measured from the first minimum of the two-point spanwise correlation of streamwise velocity, remains close to one hundred wall units at $y^+ = 15$ but grows logarithmically with distance from the wall. A simple model based on the attached-eddy hypothesis reproduces both trends and predicts the Reynolds-number dependence of the near-wall peak of the streamwise turbulence intensity. The results suggest that the inner-layer structure is universal once an outer-scaled correction is included.

Key words: turbulent boundary layers, turbulence simulation, turbulence theory

1. Introduction

Streamwise streaks of alternating high- and low-speed fluid are the most prominent feature of the near-wall region of turbulent shear flows (Kline *et al.* 1967). Their mean spanwise spacing of roughly 100 wall units is one of the best known empirical constants in wall turbulence (Smith & Metzler 1983), and it underpins both the self-sustaining process of Hamilton *et al.* (1995) and the minimal-flow-unit simulations of Jiménez & Moin (1991). Whether this spacing is truly independent of the Reynolds number, however, is still debated: as Re_τ increases, large outer-layer motions modulate the near-wall cycle (Hutchins & Marusic 2007) and may leave an imprint on the streak statistics.

The present paper addresses this question with a set of direct numerical simulations (DNS) spanning a factor of five in Re_τ . Section 2 states the governing equations and numerical method, § 3 presents the streak statistics, and § 4 compares them with an attached-eddy model. Conclusions are given in § 5.

2. Formulation and numerical method

We consider incompressible flow between two parallel plates separated by $2h$, driven by a constant mean pressure gradient. The velocity $\mathbf{u} = (u, v, w)$ and pressure p satisfy

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0, \quad (1)$$

with no-slip conditions at $y = \pm h$ and periodic conditions in x and z . Lengths are scaled with h and velocities with the bulk velocity U_b , so that $Re = U_b h / \nu$. The friction Reynolds number $Re_\tau = u_\tau h / \nu$, with $u_\tau = (\tau_w / \rho)^{1/2}$, is the control parameter of interest.

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Case	Re_τ	L_x/h	L_z/h	Δx^+	Δz^+	Tu_τ/h
C180	182	4π	2π	8.9	4.5	30
C550	547	4π	2π	8.9	4.5	15
C1000	1001	4π	2π	8.2	4.1	10

Table 1: Simulation parameters: box dimensions, grid resolution in wall units and averaging time.

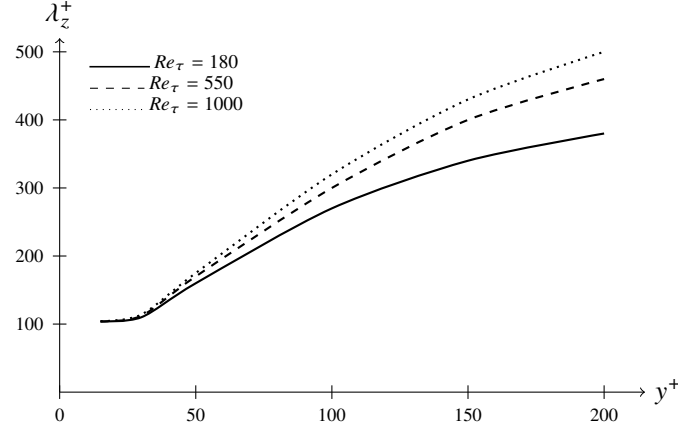


Figure 1: Spanwise streak spacing as a function of wall distance for the three Reynolds numbers of table 1.

Equations (1) are integrated with a Fourier–Chebyshev pseudo-spectral method in the wall-normal velocity–vorticity formulation of Kim *et al.* (1987), using a third-order Runge–Kutta scheme for the nonlinear terms and Crank–Nicolson for the viscous terms. Table 1 summarises the simulations.

3. Results

3.1. Streak spacing

The streak spacing λ_z^+ is defined as twice the spanwise separation at which the two-point correlation

$$R_{uu}(\Delta z; y) = \frac{\langle u'(x, y, z) u'(x, y, z + \Delta z) \rangle}{\langle u'^2 \rangle} \quad (2)$$

first reaches its minimum. Figure 1 shows λ_z^+ as a function of y^+ for the three cases. At $y^+ = 15$ the spacing is 104 ± 3 for all Reynolds numbers, confirming the classical value. Above $y^+ \approx 30$, however, the curves separate and follow $\lambda_z^+ \simeq 1.6 y^+ + 60$ up to a height that scales with h rather than with ν/u_τ .

3.2. Turbulence intensities

The near-wall peak of $\langle u'^2 \rangle^+$ grows from 7.6 at $Re_\tau = 180$ to 8.7 at $Re_\tau = 1000$, in agreement with the logarithmic growth reported by Hoyas & Jiménez (2006) and attributed to the footprint of outer motions (Hutchins & Marusic 2007).

4. An attached-eddy model

Following Townsend (1976), we represent the near-wall region as a hierarchy of eddies attached to the wall whose spanwise width is proportional to their height. A population density $P(\ell) \propto \ell^{-1}$ for $\ell_0 < \ell < h$ gives

$$\lambda_z^+(y^+) = \lambda_0^+ + \alpha y^+, \quad \langle u'^2 \rangle_{\max}^+ = B_1 - A_1 \ln\left(\frac{y^+}{Re_\tau}\right), \quad (3)$$

with $\alpha \approx 1.6$ and $A_1 \approx 1.26$ obtained from the data. The model captures both the invariant spacing at $y^+ = 15$ and the Reynolds-number growth of the peak intensity.

5. Conclusions

Near-wall streaks keep their classical spanwise spacing of about 100 wall units at $y^+ = 15$ over the Reynolds-number range studied, while their spacing farther from the wall grows linearly in y^+ in a manner consistent with the attached-eddy hypothesis. The combination of an invariant inner spacing and an outer-scaled correction accounts for the observed growth of the near-wall intensity peak. Simulations at higher Re_τ are needed to test whether the linear growth saturates.

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Declaration of interests

The authors report no conflict of interest.

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A. Derivation of the spacing model

Integrating the eddy population density over heights between y and h gives the mean width of eddies that reach height y , from which (3) follows with λ_0^+ fixed by the viscous cut-off ℓ_0 .

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